Lepton-number-violating decays of singly-charged Higgs bosons in the minimal type-(I+II) seesaw model at the TeV scale

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Abstract

The lepton-number-violating decays of singly-charged Higgs bosons $H^{\pm}$ are investigated in the minimal type-(I+II) seesaw model with one $SU(2)_L$ Higgs triplet Δ and one heavy Majorana neutrino N_1 at the TeV scale. We find that the branching ratios $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ (for $\alpha = e, \mu, \tau$) depend not only on the mass and mixing parameters of three light neutrinos ν_i (for $i = 1, 2, 3$) but also on those of N_1. Assuming the mass of N_1 to lie in the range of 200 GeV to 1 TeV, we figure out the generous interference bands for the contributions of ν_i and N_1 to $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$. We illustrate some salient features of such interference effects by considering three typical mass patterns of ν_i, and show that the relevant Majorana CP-violating phases can affect the magnitudes of $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ in this parameter region.

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I. INTRODUCTION

As the Large Hadron Collider (LHC) will soon bring us to a new energy frontier, major discoveries of new physics beyond the Standard Model (SM) at the TeV scale are highly anticipated [1]. Indeed, the observation of solar and atmospheric neutrino oscillations has provided us with the first convincing evidence for new physics beyond the SM [2]; i.e., three known neutrinos are massive and their flavors mix with one another. Whether the origin of non-zero but tiny neutrino masses can be understood at the LHC is an open but interesting question. It has recently been conjectured that possible new physics, if it exists at the TeV scale and is responsible for the electroweak symmetry breaking, might also be relevant to the neutrino mass generation [3].

The conventional seesaw picture [4], named nowadays as the type-I seesaw mechanism, gives a natural explanation of the smallness of neutrino masses by introducing a few heavy right-handed Majorana neutrinos. Another popular way to generate tiny neutrino masses, the so-called type-II seesaw mechanism, is to extend the SM by including one $SU(2)_L$ Higgs triplet [5]. One may also combine the two scenarios by assuming the existence of both the Higgs triplet and right-handed Majorana neutrinos, leading to a more general seesaw mechanism which has several different names in the literature [6]. To avoid any literal confusion, here we follow some authors and simply refer to this "hybrid" seesaw scenario as the type-(I+II) seesaw mechanism. The gauge-invariant neutrino mass terms in a type-(I+II) seesaw model can be written as

$$-\mathcal{L}_{\rm mass} \; = \; \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2} \overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + \frac{1}{2} \overline{l_{\rm L}} Y_\Delta \Delta i \sigma_2 l^c_{\rm L} + {\rm h.c.} \; , \tag{1}$$

where $M_{\rm R}$ is the mass matrix of right-handed Majorana neutrinos, and

$$\Delta \; \equiv \; \begin{pmatrix} H^- & -\sqrt{2} \; H^0 \\ \sqrt{2} \; H^{--} & -H^- \end{pmatrix} \tag{2}$$

denotes the $SU(2)_L$ Higgs triplet. After the spontaneous gauge symmetry breaking, we obtain the neutrino mass matrices $M_{\rm D} = Y_\nu v/\sqrt{2}$ and $M_{\rm L} = Y_\Delta v_\Delta$, where $\langle H \rangle \equiv v/\sqrt{2}$ and $\langle \Delta \rangle \equiv v_\Delta$ correspond to the vacuum expectation values of the neutral components of H and Δ. To minimize the degrees of freedom associated with $M_{\rm L}$, $M_{\rm D}$ and $M_{\rm R}$, one may assume that there is only one heavy Majorana neutrino (denoted as N_1) in the model with $M_{\rm R}$ and $M_{\rm D}$ being 1×1 and 3×1 respectively. Such a simplified seesaw scenario is phenomenologically viable [7–10] and can be referred to as the *minimal* type-(I+II) seesaw model, whose simplicity makes it interesting and instructive to reveal some salient features of the type-(I+II) seesaw mechanism. We shall focus our attention on this simple case in the present paper.

Our purpose is to investigate the lepton-number-violating decays of singly-charged Higgs bosons $H^\pm$ in the minimal type-(I+II) seesaw model. Such decays can naturally happen because Δ is allowed to couple to the standard-model Higgs doublet H and thus the lepton number is violated by two units [5]. If the mass scale of Δ is of $\mathcal{O}(1)$ TeV or smaller, then both $H^{\pm\pm}$ and $H^\pm$ can be produced at the LHC via the Drell-Yan process $q\bar{q} \to \gamma^*, Z^* \to H^{++} H^{--}$ and through the charged-current process $q\bar{q}' \to W^* \to H^{\pm\pm} H^\mp$. In

some optimistic scenarios, one can investigate different seesaw models by searching for the clean signals of lepton number violation in the decays of doubly- and singly-charged Higgs bosons at the TeV scale [9–13]. When it comes to large Y_Δ and small v_Δ (say, $v_\Delta < 10^{-4}$ GeV), the dominant decay channels of Δ will be the leptonic modes [12], such as $H^{++} \to l^+_\alpha l^+_\beta$ and $H^+ \to l^+_\alpha \bar{\nu}$ (for $\alpha, \beta = e, \mu, \tau$). An analysis of $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ decays in the minimal type-(I+II) seesaw model has been done in Ref. [9]. Here we are going to calculate the branching ratios of $H^+ \to l^+_\alpha \bar{\nu}$ and $H^- \to l^-_\alpha \nu$ in the same model. The importance of the lepton-number-violating decays of $H^\pm$ has been emphasized in Ref. [12] within the type-II seesaw framework. Our interest is to explore the interplay between type-I and type-II seesaw terms in $H^+ \to l^+_\alpha \bar{\nu}$ or $H^- \to l^-_\alpha \nu$ decays within the type-(I+II) seesaw framework.

Following Ref. [12], we obtain the decay rates of $H^+ \to l^+_\alpha \bar{\nu}_\beta$ as

$$\Gamma(H^+ \to l^+_\alpha \bar{\nu}_\beta) \; = \; \frac{1}{4\pi} \left| \, (Y_\Delta)_{\alpha\beta} \right|^2 M_{H^+} \; . \tag{3}$$

The branching ratios of $H^+ \to l^+_\alpha \bar{\nu}_\beta$ turn out to be [12]

$$\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu}) \equiv \sum_\beta \mathcal{B}(H^+ \to l^+_\alpha \bar{\nu}_\beta) \equiv \frac{\displaystyle\sum_\beta \Gamma(H^+ \to l^+_\alpha \bar{\nu}_\beta)}{\displaystyle\sum_{\rho,\sigma} \Gamma(H^+ \to l^+_\rho \bar{\nu}_\sigma)} = \frac{\displaystyle\sum_\beta \left| \, (M_{\rm L})_{\alpha\beta} \right|^2}{\displaystyle\sum_{\rho,\sigma} \left| \, (M_{\rm L})_{\rho\sigma} \right|^2} \; , \tag{4}$$

where the Greek subscripts run over e, μ and τ. It becomes obvious that the magnitudes of $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ are only relevant to the matrix elements of $M_{\rm L}$. Note that the matrix elements of $M_{\rm L}$ rely both on the mass and mixing parameters of three light neutrinos ν_i (for $i = 1, 2, 3$) and on those of N_1 in the minimal type-(I+II) seesaw model [9]. When the contribution of N_1 to $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ is negligibly small, our result can simply reproduce that obtained in the type-II seesaw model [12]. But when type-I and type-II seesaw terms are comparable in magnitude, we have to take care of their significant interference effects. Assuming the mass of N_1 to lie in the range of 200 GeV to 1 TeV, we figure out the generous interference bands for the contributions of ν_i and N_1 to $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$. We illustrate some salient features of such interference effects by considering three typical mass patterns of ν_i. We also show that the relevant Majorana CP-violating phases can affect the magnitudes of $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$, unlike the case in the type-II seesaw mechanism [12]. Although our numerical results are subject to the minimal type-(I+II) seesaw model, they can serve as a good example to illustrate the interplay between light and heavy Majorana neutrinos in a generic type-(I+II) seesaw scenario.

II. INTERFERENCE BANDS AND MAJORANA PHASES

After the spontaneous electroweak symmetry breaking, we rewrite Eq. (1) as

$$-\mathcal{L}^\prime_{\rm mass} \; = \; \frac{1}{2} \; \overline{(\nu_{\rm L} \;\, N^c_{\rm R})} \, \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R} \end{pmatrix} \begin{pmatrix} \nu^c_{\rm L} \\ N_{\rm R} \end{pmatrix} + {\rm h.c.} \; . \tag{5}$$

We assume the existence of only a single heavy Majorana neutrino N_1. The 4×4 neutrino mass matrix in Eq. (5) is symmetric and can be diagonalized by the following unitary transformation:

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^{\dagger} \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^{*} = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & M_1 \end{pmatrix} , \qquad (6)$$

where $\widehat{M}_\nu = {\rm Diag}\{m_1, m_2, m_3\}$ with m_i being the masses of three light neutrinos ν_i and M_1 denotes the mass of N_1. Following Ref. [14], we parametrize V and R as

$$V = \begin{pmatrix} c_{14} & 0 & 0 \\ -\hat{s}_{14}\hat{s}^*_{24} & c_{24} & 0 \\ -\hat{s}_{14}c_{24}\hat{s}^*_{34} & -\hat{s}_{24}\hat{s}^*_{34} & c_{34} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix} ,$$
$$R = \begin{pmatrix} \hat{s}^*_{14} \\ c_{14}\hat{s}^*_{24} \\ c_{14}c_{24}\hat{s}^*_{34} \end{pmatrix} , \qquad (7)$$

where $c_{ij} \equiv \theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}} s_{ij}$ with θ_{ij} and δ_{ij} (for $1 \leq i < j \leq 4$) being the rotation angles and phase angles, respectively. If the heavy Majorana neutrino N_1 is decoupled (i.e., $\theta_{14} = \theta_{24} = \theta_{34} = 0$), V will become a unitary matrix and take the standard form [2]. Hence non-vanishing R measures the non-unitarity of V.

Now we make use of Eqs. (6) and (7) to reconstruct the matrix elements of $M_{\rm L}$ in terms of m_i, M_1, V and R. It is easy to obtain $M_{\rm L} = V\widehat{M}_\nu V^T + M_1 RR^T$. Taking the approximation $c_{13} \approx c_{i4} \approx 1$ based on current experimental constraints $s_{13} < 0.16$ [15] and $s_{i4} \lesssim 0.1$ (for $i = 1, 2, 3$) [16], we arrive at

$$\begin{aligned}
\sum_\beta |\,(M_{\rm L})_{e\beta}\,|^2 = {} & m_1^2 c_{12}^2 + m_2^2 s_{12}^2 + m_3^2 s_{13}^2 + M_1 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
& + 2m_1 M_1 {\rm Re}\left[c_{12}\hat{s}_{14}\left(c_{12}\hat{s}_{14} - \hat{s}_{12}c_{23}\hat{s}_{24} + \hat{s}_{12}\hat{s}_{23}\hat{s}_{34}\right)\right] \\
& + 2m_2 M_1 {\rm Re}\left[\hat{s}^*_{12}\hat{s}_{14}\left(\hat{s}^*_{12}\hat{s}_{14} + c_{12}c_{23}\hat{s}_{24} - c_{12}\hat{s}_{23}\hat{s}_{34}\right)\right] \\
& + 2m_3 M_1 {\rm Re}\left[\hat{s}^*_{13}\hat{s}_{14}\left(\hat{s}^*_{13}\hat{s}_{14} + \hat{s}^*_{23}\hat{s}_{24} + c_{23}\hat{s}_{34}\right)\right] , \\
\sum_\beta |\,(M_{\rm L})_{\mu\beta}\,|^2 = {} & m_1^2 s_{12}^2 c_{23}^2 + m_2^2 c_{12}^2 c_{23}^2 + m_3^2 s_{23}^2 + M_1 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
& - 2m_1 M_1 {\rm Re}\left[\hat{s}_{12}c_{23}\hat{s}_{24}\left(c_{12}\hat{s}_{14} - \hat{s}_{12}c_{23}\hat{s}_{24} + \hat{s}_{12}\hat{s}_{23}\hat{s}_{34}\right)\right] \\
& + 2m_2 M_1 {\rm Re}\left[c_{12}c_{23}\hat{s}_{24}\left(\hat{s}^*_{12}\hat{s}_{14} + c_{12}c_{23}\hat{s}_{24} - c_{12}\hat{s}_{23}\hat{s}_{34}\right)\right] \\
& + 2m_3 M_1 {\rm Re}\left[\hat{s}^*_{23}\hat{s}_{24}\left(\hat{s}^*_{13}\hat{s}_{14} + \hat{s}^*_{23}\hat{s}_{24} + c_{23}\hat{s}_{34}\right)\right] , \\
\sum_\beta |\,(M_{\rm L})_{\tau\beta}\,|^2 = {} & m_1^2 s_{12}^2 s_{23}^2 + m_2^2 c_{12}^2 s_{23}^2 + m_3^2 c_{23}^2 + M_1 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
& + 2m_1 M_1 {\rm Re}\left[\hat{s}_{12}\hat{s}_{23}\hat{s}_{34}\left(c_{12}\hat{s}_{14} - \hat{s}_{12}c_{23}\hat{s}_{24} + \hat{s}_{12}\hat{s}_{23}\hat{s}_{34}\right)\right] \\
& - 2m_2 M_1 {\rm Re}\left[c_{12}\hat{s}_{23}\hat{s}_{34}\left(\hat{s}^*_{12}\hat{s}_{14} + c_{12}c_{23}\hat{s}_{24} - c_{12}\hat{s}_{23}\hat{s}_{34}\right)\right] \\
& + 2m_3 M_1 {\rm Re}\left[c_{23}\hat{s}_{34}\left(\hat{s}^*_{13}\hat{s}_{14} + \hat{s}^*_{23}\hat{s}_{24} + c_{23}\hat{s}_{34}\right)\right] ;
\end{aligned} \qquad (8)$$

and

$$\begin{aligned}
\sum_{\rho,\sigma} |\,(M_{\rm L})_{\rho\sigma}\,|^2 = {} & \left(m_1^2 + m_2^2 + m_3^2\right) + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 \\
& + 2m_1 M_1 {\rm Re}\left[\left(c_{12}\hat{s}_{14} - \hat{s}_{12}c_{23}\hat{s}_{24} + \hat{s}_{12}\hat{s}_{23}\hat{s}_{34}\right)^2\right] \\
& + 2m_2 M_1 {\rm Re}\left[\left(\hat{s}^*_{12}\hat{s}_{14} + c_{12}c_{23}\hat{s}_{24} - c_{12}\hat{s}_{23}\hat{s}_{34}\right)^2\right] \\
& + 2m_3 M_1 {\rm Re}\left[\left(\hat{s}^*_{13}\hat{s}_{14} + \hat{s}^*_{23}\hat{s}_{24} + c_{23}\hat{s}_{34}\right)^2\right] .
\end{aligned} \qquad (9)$$

By combining Eqs. (8) and (9) with Eq. (4), we are then able to calculate the branching ratios $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$. Note that these branching ratios can also be expressed in terms of the branching ratios $\mathcal{B}(H^{++} \to l^+_\alpha l^+_\beta)$ obtained in Ref. [9]; namely,

$$
\begin{aligned}
\mathcal{B}(H^+ \to e^+ \bar{\nu}) &= \mathcal{B}(H^{++} \to e^+ e^+) + \frac{1}{2}\left[\mathcal{B}(H^{++} \to e^+ \mu^+) + \mathcal{B}(H^{++} \to e^+ \tau^+)\right] \; , \\
\mathcal{B}(H^+ \to \mu^+ \bar{\nu}) &= \mathcal{B}(H^{++} \to \mu^+ \mu^+) + \frac{1}{2}\left[\mathcal{B}(H^{++} \to e^+ \mu^+) + \mathcal{B}(H^{++} \to \mu^+ \tau^+)\right] \; , \\
\mathcal{B}(H^+ \to \tau^+ \bar{\nu}) &= \mathcal{B}(H^{++} \to \tau^+ \tau^+) + \frac{1}{2}\left[\mathcal{B}(H^{++} \to e^+ \tau^+) + \mathcal{B}(H^{++} \to \mu^+ \tau^+)\right] \; . \qquad (10)
\end{aligned}
$$

If the heavy Majorana neutrino N_1 is essentially decoupled (i.e., $\theta_{i4} \approx 0$ for $i = 1, 2, 3$), then the unitarity of V will be restored. In this case, the results of $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ are the same as those given in the type-II seesaw model [12].

If the contributions of ν_i and N_1 to $(M_{\rm L})_{\alpha\beta}$ are comparable in magnitude, there will be significant interference effects on the branching ratios of $H^+ \to l^+_\alpha \bar{\nu}$ decays. To be explicit, we take $\Delta m^2_{21} \sim 7.7 \times 10^{-5}$ eV^2 and $|\Delta m^2_{32}| \sim 2.4 \times 10^{-3}$ eV^2 [15] as the typical inputs and assume M_1 to lie in the range of 200 GeV to 1 TeV. There are three possible patterns of the light neutrino mass spectrum: (1) the normal hierarchy: $m_3 \sim 5.0 \times 10^{-2}$ eV, $m_2 \sim 8.8 \times 10^{-3}$ eV, and m_1 is much smaller than m_2; (2) the inverted hierarchy: $m_2 \sim 4.9 \times 10^{-2}$ eV, $m_1 \sim 4.8 \times 10^{-2}$ eV, and m_3 is much smaller than m_1; (3) the near degeneracy: $m_1 \sim m_2 \sim m_3 \sim 0.1$ eV to 0.2 eV, which is consistent with the cosmological upper bound $m_1 + m_2 + m_3 < 0.67$ eV [17]. In each case, the contributions of ν_i and N_1 to $(M_{\rm L})_{\alpha\beta}$ in Eq. (8) will be of the comparable magnitude if the mixing angles θ_{i4} satisfy the condition [9]

$$
s_{i4} s_{j4} \; \sim \; \frac{\max\{m_1, m_2, m_3\}}{M_1} \; \sim \; 10^{-14} \cdots 10^{-12} \; , \qquad (11)
$$

where $i, j = 1, 2, 3$. This rough estimate allows us to set $\sqrt{s_{i4} s_{j4}} \sim 10^{-8}$—$10^{-5}$ as the interference bands of $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ for M_1 to vary between 200 GeV and 1 TeV. Because the CP-violating phases δ_{i4} are completely unrestricted, they may cause either constructive or destructive effects in the interference bands.

To see the impacts of the Majorana phases on the branching ratios $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ in this arresting parameter region, we may properly redefine the phases of three charged-lepton fields and then reexpress the neutrino mixing matrix V in Eq. (7) as

$$
V \; = \; \begin{pmatrix} c_{14} & 0 & 0 \\ -s_{14} s_{24} e^{i\phi} & c_{24} & 0 \\ -s_{14} c_{24} s_{34} e^{i(\phi + \varphi)} & -s_{24} s_{34} e^{i\varphi} & c_{34} \end{pmatrix} V_0 \; , \qquad (12)
$$

where

$$
V_0 \; = \; \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{13} s_{23} e^{i\delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta} & c_{13} s_{23} \\ s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta} & -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{i\delta} & c_{13} c_{23} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (13)
$$

denotes the standard parametrization of the 3×3 unitary neutrino mixing matrix [2], and the relevant CP-violating phases are defined as $\phi = \delta_{14} - \delta_{24} - \delta_{12}$, $\varphi = \delta_{24} - \delta_{34} - \delta_{23}$,

$\delta = \delta_{13} - \delta_{12} - \delta_{23}$, $\rho = \delta_{12} + \delta_{23}$ and $\sigma = \delta_{23}$. It is clear that ρ and σ are the so-called Majorana phases because they have nothing to do with neutrino oscillations but may affect the neutrinoless double-beta decay. With the help of Eqs. (12) and (13), we may rewrite Eqs. (8) and (9) as follows:

$$\begin{aligned}
\sum_\beta |\left(M_{\rm L}\right)_{e\beta}|^2 = \; & m_1^2 c_{12}^2 + m_2^2 s_{12}^2 + m_3^2 s_{13}^2 + M_1^2 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
& + 2m_1 M_1 {\rm Re}\left[c_{12} s_{14} e^{2i\delta_{14}} \left(c_{12} s_{14} - s_{12} c_{23} s_{24} e^{-i\phi} + s_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right] \\
& + 2m_2 M_1 {\rm Re}\left[s_{12} s_{14} e^{2i(\delta_{14}-\rho+\sigma)} \left(s_{12} s_{14} + c_{12} c_{23} s_{24} e^{-i\phi} - c_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right] \\
& + 2m_3 M_1 {\rm Re}\left[s_{13} s_{14} e^{i(2\delta_{14}-2\rho-\delta-\phi-\varphi)} \left(s_{13} s_{14} e^{i(\phi+\varphi-\delta)} + s_{23} s_{24} e^{i\varphi} + c_{23} s_{34}\right)\right] \; , \\
\sum_\beta |\left(M_{\rm L}\right)_{\mu\beta}|^2 = \; & m_1^2 s_{12}^2 c_{23}^2 + m_2^2 c_{12}^2 c_{23}^2 + m_3^2 s_{23}^2 + M_1^2 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
& - 2m_1 M_1 {\rm Re}\left[s_{12} c_{23} s_{24} e^{i(2\delta_{14}-\phi)} \left(c_{12} s_{14} - s_{12} c_{23} s_{24} e^{-i\phi} + s_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right] \\
& + 2m_2 M_1 {\rm Re}\left[c_{12} c_{23} s_{24} e^{i(2\delta_{14}-2\rho+2\sigma-\phi)} \left(s_{12} s_{14} + c_{12} c_{23} s_{24} e^{-i\phi} - c_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right] \\
& + 2m_3 M_1 {\rm Re}\left[s_{23} s_{24} e^{i(2\delta_{14}-2\rho-2\phi-\varphi)} \left(s_{13} s_{14} e^{i(\phi+\varphi-\delta)} + s_{23} s_{24} e^{i\varphi} + c_{23} s_{34}\right)\right] \; , \\
\sum_\beta |\left(M_{\rm L}\right)_{\tau\beta}|^2 = \; & m_1^2 s_{12}^2 s_{23}^2 + m_2^2 c_{12}^2 s_{23}^2 + m_3^2 c_{23}^2 + M_1^2 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
& + 2m_1 M_1 {\rm Re}\left[s_{12} s_{23} s_{34} e^{i(2\delta_{14}-\phi-\varphi)} \left(c_{12} s_{14} - s_{12} c_{23} s_{24} e^{-i\phi} + s_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right] \\
& - 2m_2 M_1 {\rm Re}\left[c_{12} s_{23} s_{34} e^{i(2\delta_{14}-2\rho+2\sigma-\phi-\varphi)} \left(s_{12} s_{14} + c_{12} c_{23} s_{24} e^{-i\phi} - c_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right] \\
& + 2m_3 M_1 {\rm Re}\left[c_{23} s_{34} e^{2i(\delta_{14}-\rho-\phi-\varphi)} \left(s_{13} s_{14} e^{i(\phi+\varphi-\delta)} + s_{23} s_{24} e^{i\varphi} + c_{23} s_{34}\right)\right] \; , \\
\sum_{\rho,\sigma} |\left(M_{\rm L}\right)_{\rho\sigma}|^2 = \; & m_1^2 + m_2^2 + m_3^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 \\
& + 2m_1 M_1 {\rm Re}\left[e^{i\delta_{14}} \left(c_{12} s_{14} - s_{12} c_{23} s_{24} e^{-i\phi} + s_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right]^2 \\
& + 2m_2 M_1 {\rm Re}\left[e^{i(\delta_{14}-\rho+\sigma)} \left(s_{12} s_{14} + c_{12} c_{23} s_{24} e^{-i\phi} - c_{12} s_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right]^2 \\
& + 2m_3 M_1 {\rm Re}\left[e^{i(\delta_{14}-\rho)} \left(s_{13} s_{14} e^{-i\delta} + s_{23} s_{24} e^{-i\phi} + c_{23} s_{34} e^{-i(\phi+\varphi)}\right)\right]^2 \; .
\end{aligned} \tag{14}$$

We see that the conventional Majorana phases ρ and σ together with other CP-violating phases show up in the interference terms. Hence they may affect the branching ratios of $H^+ \to l^+_\alpha \bar{\nu}$ decays to some extent. We shall numerically calculate $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$ in the subsequent section to illustrate both the interference bands and the effects of Majorana phases for different mass spectra of three light neutrinos.

If $M_1 \lesssim \mathcal{O}(1)$ TeV and the values of s_{i4} lie in the interference bands obtained above, it will be impossible to produce and observe N_1 at the LHC. The reason is simply that the interaction of N_1 with three charged leptons is too weak to be detected in this parameter space [9]. Given the integrated luminosity to be 100 fb^{-1}, for example, the resonant signature of N_1 in the channel $p\bar{p} \to \mu^\pm N_1$ with $N_1 \to \mu^\pm W^\mp$ at the LHC has been analyzed and the sensitivity of the cross section $\sigma(p\bar{p} \to \mu^\pm \mu^\pm W^\mp) \approx \sigma(p\bar{p} \to \mu^\pm N_1)\mathcal{B}(N_1 \to \mu^\pm W^\mp)$ to the effective mixing parameter $S_{\mu\mu} \approx s_{24}^4/(s_{14}^2 + s_{24}^2 + s_{34}^2)$ has been examined in Ref. [18]. It is found that $S_{\mu\mu} \geq 7.2 \times 10^{-4}$ (or equivalently, $s_{24}^2 \geq 2.1 \times 10^{-3}$ for $s_{14} \sim s_{24} \sim s_{34}$) is required in order to get a signature at the 2σ level for $M_1 \geq 200$ GeV. This result illustrates

that there will be no chance to probe the existence of N_1 in the interference bands at the LHC. However, it is possible to produce $H^\pm$ and $H^{\pm\pm}$ at the LHC and to observe the signatures of $H^+ \to l^+_\alpha \bar{\nu}$, $H^- \to l^-_\alpha \nu$ and $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ decays provided $M_{H^\pm} \lesssim \mathcal{O}(1)$ TeV and $M_{H^{\pm\pm}} \lesssim \mathcal{O}(1)$ TeV [12]. In this case, the measurements of relevant decay rates or branching ratios are difficult to tell whether the existence of $H^\pm$ and $H^{\pm\pm}$ is due to a pure type-II seesaw model or due to a (minimal) type-(I+II) seesaw model.

III. NUMERICAL EXAMPLES

For the sake of simplicity, here we take $\theta_{12} = \arctan(1/\sqrt{2}) \approx 35.3°$, $\theta_{13} = 0°$ and $\theta_{23} = 45°$; i.e., V_0 takes the exact tri-bimaximal mixing pattern [19]. The small deviation of V from V_0 implies the effect of unitarity violation. We shall do the numerical calculations in two different ways. Firstly, to examine the nontrivial role of new CP-violating phases δ_{i4} in $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$, we switch off the conventional CP-violating phases δ_{12}, δ_{13} and δ_{23}. We fix $\Delta m^2_{21} = 7.7 \times 10^{-5}$ eV^2, $|\Delta m^2_{32}| = 2.4 \times 10^{-3}$ eV^2 and $M_1 = 500$ GeV in our calculations. To further reduce the number of free parameters, we shall consider one special case for the mixing angles θ_{i4} (e.g., $\theta_{14} = \theta_{24} = \theta_{34}$) and two special cases for the CP-violating phases δ_{i4} (either $\delta_{14} = \delta_{24} = \delta_{34} = 0$ or $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$). Secondly, to illustrate the remarkable effects of two conventional Majorana phases ρ and σ on $\mathcal{B}(H^+ \to l^+_\alpha \bar{\nu})$, we switch off other CP-violating phases and take $\theta \equiv \theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ as a typical input within the interference bands. Our results and discussions can be classified into three parts in accordance with three possible mass patterns of three light neutrinos.

A. Normal hierarchy

We simply take $m_1 = 0$, such that $m_2 \approx 8.8 \times 10^{-3}$ eV and $m_3 \approx 5.0 \times 10^{-2}$ eV can be extracted from the given values of Δm^2_{21} and $|\Delta m^2_{32}|$. For chosen values of θ_{12}, θ_{13} and θ_{23} together with the assumption $\delta_{12} = \delta_{13} = \delta_{23} = 0$, Eqs. (8) and (9) can now be simplified to

$$\begin{aligned}
\sum_\beta |\,(M_{\rm L})_{e\beta}\,|^2 &= \frac{1}{3}m_2^2 + M_1^2 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + \frac{2}{3}m_2 M_1 {\rm Re}\left[\hat{s}_{14}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)\right] \ , \\
\sum_\beta |\,(M_{\rm L})_{\mu\beta}\,|^2 &= \frac{1}{3}m_2^2 + \frac{1}{2}m_3^2 + M_1^2 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
&\quad + \frac{2}{3}m_2 M_1 {\rm Re}\left[\hat{s}_{24}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)\right] + m_3 M_1 {\rm Re}\left[\hat{s}_{24}\left(\hat{s}_{24} + \hat{s}_{34}\right)\right] \ , \\
\sum_\beta |\,(M_{\rm L})_{\tau\beta}\,|^2 &= \frac{1}{3}m_2^2 + \frac{1}{2}m_3^2 + M_1^2 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
&\quad - \frac{2}{3}m_2 M_1 {\rm Re}\left[\hat{s}_{34}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)\right] + m_3 M_1 {\rm Re}\left[\hat{s}_{34}\left(\hat{s}_{24} + \hat{s}_{34}\right)\right] \ , \\
\sum_{\rho,\sigma} |\,(M_{\rm L})_{\rho\sigma}\,|^2 &= m_2^2 + m_3^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 \\
&\quad + \frac{2}{3}m_2 M_1 {\rm Re}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)^2 + m_3 M_1 {\rm Re}\left(\hat{s}_{24} + \hat{s}_{34}\right)^2 \ . \qquad (15)
\end{aligned}$$

Our numerical results for the branching ratios $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ are shown in FIG. 1(a) and FIG. 1(b).

FIG. 1(a) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$. We see that $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ and $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ are approximately the same in the whole parameter space due to an approximate μ-τ symmetry.

FIG. 1(b) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$. We see more obvious interference effects for θ changing from 10^{-7} to 10^{-6}, which can be understood with the help of Eqs. (4) and (15). In particular, $\mathcal{B}(H^+ \to e^+ \bar{\nu})$ is strongly enhanced because of the destructive interference effect in its denominator, while $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ and $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ may reach their minimal values due to the destructive interference effects in their numerators at $\theta \sim 2 \times 10^{-7}$.

On the other hand, let us simplify Eq. (14) by taking $\delta_{14} = \phi = \varphi = \delta = 0$:

$$
\begin{aligned}
\sum_\beta |\, (M_{\rm L})_{e\beta}\,|^2 &= \frac{1}{3} m_2^2 + M_1^2 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + \frac{2}{3} m_2 M_1 s_{14} \left(s_{14} + s_{24} - s_{34}\right) \cos 2(\rho - \sigma) \;, \\
\sum_\beta |\, (M_{\rm L})_{\mu\beta}\,|^2 &= \frac{1}{3} m_2^2 + \frac{1}{2} m_3^2 + M_1^2 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
&\quad + \frac{2}{3} m_2 M_1 s_{24} \left(s_{14} + s_{24} - s_{34}\right) \cos 2(\rho - \sigma) + m_3 M_1 s_{24} \left(s_{24} + s_{34}\right) \cos 2\rho \;, \\
\sum_\beta |\, (M_{\rm L})_{\tau\beta}\,|^2 &= \frac{1}{3} m_2^2 + \frac{1}{2} m_3^2 + M_1^2 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
&\quad - \frac{2}{3} m_2 M_1 s_{34} \left(s_{14} + s_{24} - s_{34}\right) \cos 2(\rho - \sigma) + m_3 M_1 s_{34} \left(s_{24} + s_{34}\right) \cos 2\rho \;, \\
\sum_{\rho,\sigma} |\, (M_{\rm L})_{\rho\sigma}\,|^2 &= m_2^2 + m_3^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 \\
&\quad + \frac{2}{3} m_2 M_1 \left(s_{14} + s_{24} - s_{34}\right)^2 \cos 2(\rho - \sigma) + m_3 M_1 \left(s_{24} + s_{34}\right)^2 \cos 2\rho \;. \qquad (16)
\end{aligned}
$$

Our numerical results for the branching ratios $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ are shown in FIG. 1(c) and FIG. 1(d).

FIG. 1(c) is obtained by taking both $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\sigma = \delta_{14} = \phi = \varphi = \delta = 0$. We see that $\mathcal{B}(H^+ \to e^+ \bar{\nu})$, $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ and $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ are all sensitive to the Majorana phase ρ changing from 0 to 2π.

FIG. 1(d) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\rho = \delta_{14} = \phi = \varphi = \delta = 0$. The slight difference between $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ and $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ is easily understandable from Eq. (16). Compared with FIG. 1(c), FIG. 1(d) reveals a rather mild dependence of $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ on the Majorana phase σ. The reason is simply that the terms proportional to $\cos 2(\rho - \sigma)$ are more suppressed than those proportional to $\cos 2\rho$ in Eq. (16), as a straightforward result of $m_2 < m_3$.

B. Inverted hierarchy

We take $m_3 = 0$ for simplicity, such that $m_1 \approx 4.8 \times 10^{-2}$ eV and $m_2 \approx 4.9 \times 10^{-2}$ eV can be extracted from the given values of Δm_{21}^2 and $|\Delta m_{32}^2|$. For chosen values of θ_{12}, θ_{13}

and θ_{23} together with the assumption $\delta_{12} = \delta_{13} = \delta_{23} = 0$, Eqs. (8) and (9) can now be simplified to

$$\begin{aligned}
\sum_\beta |\,(M_{\rm L})_{e\beta}\,|^2 &= \frac{2}{3}m_1^2 + \frac{1}{3}m_2^2 + M_1^2 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
&\quad + \frac{2}{3}m_1 M_1 {\rm Re}\left[\hat{s}_{14}\left(2\hat{s}_{14} - \hat{s}_{24} + \hat{s}_{34}\right)\right] + \frac{2}{3}m_2 M_1 {\rm Re}\left[\hat{s}_{14}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)\right] \ , \\
\sum_\beta |\,(M_{\rm L})_{\mu\beta}\,|^2 &= \frac{1}{6}m_1^2 + \frac{1}{3}m_2^2 + M_1^2 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
&\quad - \frac{1}{3}m_1 M_1 {\rm Re}\left[\hat{s}_{24}\left(2\hat{s}_{14} - \hat{s}_{24} + \hat{s}_{34}\right)\right] + \frac{2}{3}m_2 M_1 {\rm Re}\left[\hat{s}_{24}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)\right] \ , \\
\sum_\beta |\,(M_{\rm L})_{\tau\beta}\,|^2 &= \frac{1}{6}m_1^2 + \frac{1}{3}m_2^2 + M_1^2 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) \\
&\quad + \frac{1}{3}m_1 M_1 {\rm Re}\left[\hat{s}_{34}\left(2\hat{s}_{14} - \hat{s}_{24} + \hat{s}_{34}\right)\right] - \frac{2}{3}m_2 M_1 {\rm Re}\left[\hat{s}_{34}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)\right] \ , \\
\sum_{\rho,\sigma} |\,(M_{\rm L})_{\rho\sigma}\,|^2 &= m_1^2 + m_2^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 \\
&\quad + \frac{1}{3}m_1 M_1 {\rm Re}\left(2\hat{s}_{14} - \hat{s}_{24} + \hat{s}_{34}\right)^2 + \frac{2}{3}m_2 M_1 {\rm Re}\left(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34}\right)^2 \ . \qquad (17)
\end{aligned}$$

Our numerical results for the branching ratios $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ are shown in FIG. 2(a) and FIG. 2(b).

FIG. 2(a) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$. We see that $\mathcal{B}(H^+ \to \mu^+ \bar{\nu}) = \mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ holds in the whole parameter space due to μ-τ symmetry.

FIG. 2(b) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$. One can see more obvious interference effects for θ changing from 10^{-7} to 10^{-6}, which can be understood with the help of Eqs. (4) and (17). In particular, $\mathcal{B}(H^+ \to e^+ \bar{\nu})$ undergoes a minimum because of the destructive interference effect in its numerator, while $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ or $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ undergoes a maximum due to the destructive interference effect in its denominator when θ varies in the interference band.

On the other hand, we simplify Eq. (14) by taking $\delta_{14} = \phi = \varphi = \delta = 0$:

$$\begin{aligned}
\sum_\beta |\,(M_{\rm L})_{e\beta}\,|^2 &= \frac{2}{3}m_1^2 + \frac{1}{3}m_2^2 + M_1^2 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + \frac{2}{3}m_1 M_1 s_{14}\left(2s_{14} - s_{24} + s_{34}\right) \\
&\quad + \frac{2}{3}m_2 M_1 s_{14}\left(s_{14} + s_{24} - s_{34}\right)\cos 2(\rho - \sigma) \ , \\
\sum_\beta |\,(M_{\rm L})_{\mu\beta}\,|^2 &= \frac{1}{6}m_1^2 + \frac{1}{3}m_2^2 + M_1^2 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) - \frac{1}{3}m_1 M_1 s_{24}\left(2s_{14} - s_{24} + s_{34}\right) \\
&\quad + \frac{2}{3}m_2 M_1 s_{24}\left(s_{14} + s_{24} - s_{34}\right)\cos 2(\rho - \sigma) \ , \\
\sum_\beta |\,(M_{\rm L})_{\tau\beta}\,|^2 &= \frac{1}{6}m_1^2 + \frac{1}{3}m_2^2 + M_1^2 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + \frac{1}{3}m_1 M_1 s_{34}\left(2s_{14} - s_{24} + s_{34}\right) \\
&\quad - \frac{2}{3}m_2 M_1 s_{34}\left(s_{14} + s_{24} - s_{34}\right)\cos 2(\rho - \sigma) \ ,
\end{aligned}$$

$$\sum_{\rho,\sigma} |\,(M_{\rm L})_{\rho\sigma}\,|^2 = m_1^2 + m_2^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 + \frac{1}{3} m_1 M_1 \left(2s_{14} - s_{24} + s_{34}\right)^2$$
$$+ \frac{2}{3} m_2 M_1 \left(s_{14} + s_{24} - s_{34}\right)^2 \cos 2(\rho - \sigma) \; . \qquad (18)$$

Our numerical results for the branching ratios $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ are shown in FIG. 2(c) and FIG. 2(d).

FIG. 2(c) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\sigma = \delta_{14} = \phi = \varphi = \delta = 0$. We see that $\mathcal{B}(H^+ \to e^+ \bar{\nu})$, $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ and $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ are all sensitive to the Majorana phase ρ varying from 0 to 2π. FIG. 2(d) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\rho = \delta_{14} = \phi = \varphi = \delta = 0$. Hence the results of $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ in FIG. 2(d) are the same as those in FIG. 2(c), as a straightforward consequence of the ρ-σ permutation symmetry which can be seen from Eq. (18).

C. Near degeneracy

We assume $m_1 \approx m_2 \approx m_3 \approx 0.1$ eV. Then $m_2 - m_1 \approx 3.9 \times 10^{-4}$ eV and $m_3 - m_2 \approx \pm 1.2 \times 10^{-2}$ eV can be extracted from given values of Δm_{21}^2 and $|\Delta m_{32}^2|$, respectively. For chosen values of θ_{12}, θ_{13} and θ_{23} together with the assumption $\delta_{12} = \delta_{13} = \delta_{23} = 0$, Eqs. (8) and (9) can now be simplified to

$$\sum_{\beta} |\,(M_{\rm L})_{e\beta}\,|^2 \approx m_1^2 + M_1^2 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + 2 m_1 M_1 s_{14}^2 \;\cos 2\delta_{14} \; ,$$
$$\sum_{\beta} |\,(M_{\rm L})_{\mu\beta}\,|^2 \approx m_1^2 + M_1^2 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + 2 m_1 M_1 s_{24}^2 \;\cos 2\delta_{24} \; ,$$
$$\sum_{\beta} |\,(M_{\rm L})_{\tau\beta}\,|^2 \approx m_1^2 + M_1^2 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + 2 m_1 M_1 s_{34}^2 \;\cos 2\delta_{34} \; ,$$
$$\sum_{\rho,\sigma} |\,(M_{\rm L})_{\rho\sigma}\,|^2 \approx 3 m_1^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2$$
$$+ 2 m_1 M_1 \left(s_{14}^2 \;\cos 2\delta_{14} + s_{24}^2 \;\cos 2\delta_{24} + s_{34}^2 \;\cos 2\delta_{34}\right) \; , \qquad (19)$$

where we have omitted the small mass differences of ν_i. We fix $m_3 > m_2$ and keep two small mass differences in our numerical calculations. The results for $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ are shown in FIG. 3(a) and FIG. 3(b).

FIG. 3(a) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$. We find that $\mathcal{B}(H^+ \to e^+ \bar{\nu}) \approx \mathcal{B}(H^+ \to \mu^+ \bar{\nu}) \approx \mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ approximately holds in the whole parameter space, as one can simply see from Eq. (19). Similar results are also obtained in FIG. 3(b), where $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$ have been taken. In both cases, the changes of $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ with θ are very mild.

On the other hand, we simplify Eq. (14) by taking $\delta_{14} = \phi = \varphi = \delta = 0$:

$$\sum_{\beta} |\,(M_{\rm L})_{e\beta}\,|^2 \approx m_1^2 + M_1^2 s_{14}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + \frac{2}{3} m_1 M_1 s_{14} \left[(2s_{14} - s_{24} + s_{34})\right.$$
$$\left. + \left(s_{14} + s_{24} - s_{34}\right) \cos 2(\rho - \sigma)\right] \; ,$$

$$
\begin{aligned}
\sum_{\beta} |\,(M_{\rm L})_{\mu\beta}\,|^2 \approx\; & m_1^2 + M_1^2 s_{24}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) - \frac{1}{3} m_1 M_1 s_{24} \left[(2s_{14} - s_{24} + s_{34})\right. \\
& \left. -2\,(s_{14} + s_{24} - s_{34}) \cos 2(\rho - \sigma) - 3\,(s_{24} + s_{34}) \cos 2\rho\right] \ , \\
\sum_{\beta} |\,(M_{\rm L})_{\tau\beta}\,|^2 \approx\; & m_1^2 + M_1^2 s_{34}^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) + \frac{1}{3} m_1 M_1 s_{34} \left[(2s_{14} - s_{24} + s_{34})\right. \\
& \left. -2\,(s_{14} + s_{24} - s_{34}) \cos 2(\rho - \sigma) + 3\,(s_{24} + s_{34}) \cos 2\rho\right] \ , \\
\sum_{\rho,\sigma} |\,(M_{\rm L})_{\rho\sigma}\,|^2 \approx\; & 3m_1^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 + \frac{1}{3} m_1 M_1 \left[(2s_{14} - s_{24} + s_{34})^2\right. \\
& \left. +2\,(s_{14} + s_{24} - s_{34})^2 \cos 2(\rho - \sigma) + 3\,(s_{24} + s_{34})^2 \cos 2\rho\right] \ , \qquad (20)
\end{aligned}
$$

where we have omitted the small mass differences of ν_i. We fix $m_3 > m_2$ and keep two small mass differences in our numerical calculations. The results for $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ are shown in FIG. 3(c) and FIG. 3(d).

FIG. 3(c) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\sigma = \delta_{14} = \phi = \varphi = \delta = 0$. We see that $\mathcal{B}(H^+ \to e^+ \bar{\nu})$, $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ and $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ are all sensitive to the Majorana phase ρ changing from 0 to 2π.

FIG. 3(d) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\rho = \delta_{14} = \phi = \varphi = \delta = 0$. We see that the behaviors of $\mathcal{B}(H^+ \to e^+ \bar{\nu})$, $\mathcal{B}(H^+ \to \mu^+ \bar{\nu})$ and $\mathcal{B}(H^+ \to \tau^+ \bar{\nu})$ changing with the Majorana phase σ are different from and milder than those in FIG. 3(c), as one can easily understand from Eq. (20).

IV. SUMMARY

We have studied the lepton-number-violating decays of singly-charged Higgs bosons $H^\pm$ in the minimal type-(I+II) seesaw model with one heavy Majorana neutrino N_1 and one $SU(2)_L$ Higgs triplet Δ at the TeV scale. Their branching ratios $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$ depend not only on the masses, flavor mixing angles and CP-violating phases of three light neutrinos ν_i (for $i = 1, 2, 3$) but also on those of N_1. We have focused our attention on the interference bands of $\mathcal{B}(H^+ \to l_\alpha^+ \bar{\nu})$, in which the contributions of light and heavy Majorana neutrinos are comparable in magnitude. We emphasize that both constructive and destructive interference effects are possible in the interference bands, and thus it is very difficult to distinguish the (minimal) type-(I+II) seesaw model from the type-II seesaw model in this parameter space. While the lepton-number-violating decays of $H^\pm$ are independent of the conventional Majorana phases ρ and σ in the type-II seesaw mechanism, they *do* depend on ρ and σ in the type-(I+II) seesaw scenario. Although our numerical results are subject to a simplified type-(I+II) seesaw model, they can serve as a good example to illustrate the interplay between type-I and type-II seesaw terms in a generic type-(I+II) seesaw framework which involves more free parameters.

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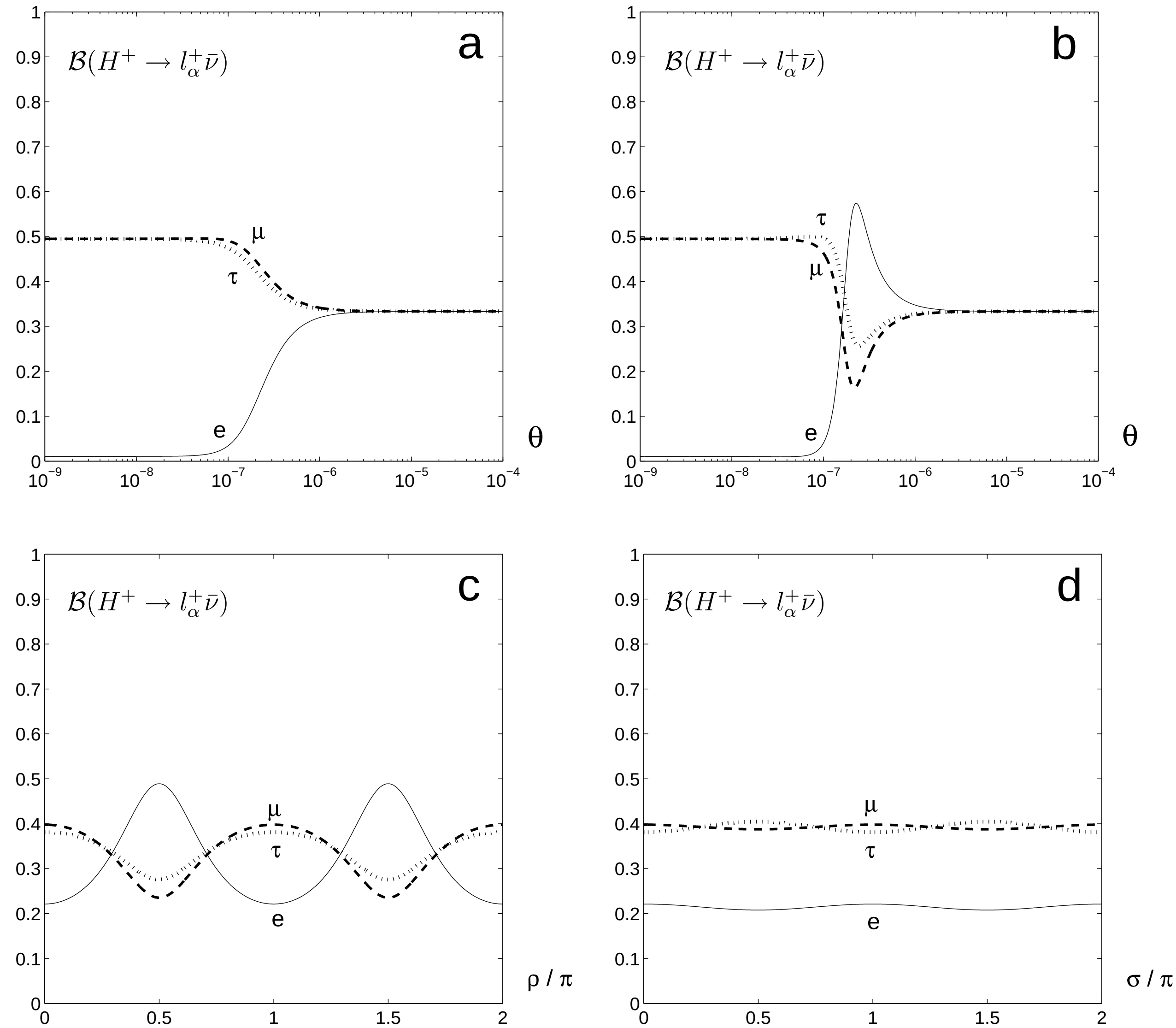

FIG. 1. Branching ratios of $H^+ \to l_\alpha^+ \bar{\nu}$ decays for the normal hierarchy of m_i with $m_1 = 0$: (a) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$; (b) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$; (c) $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\delta_{14} = \phi = \varphi = \delta = \sigma = 0$; (d) $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\delta_{14} = \phi = \varphi = \delta = \rho = 0$.

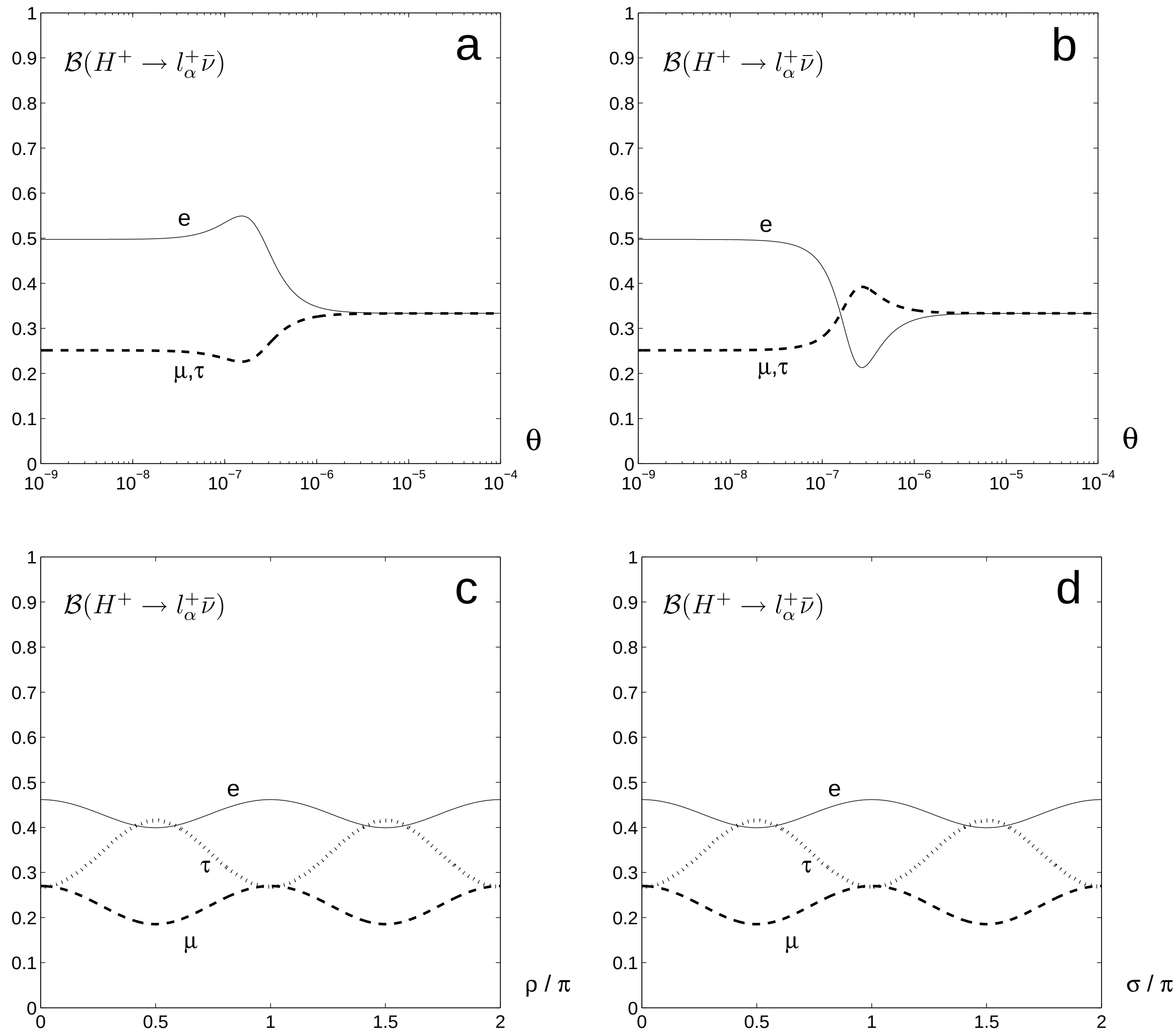

FIG. 2. Branching ratios of $H^+ \to l_\alpha^+ \bar{\nu}$ decays for the inverted hierarchy of m_i with $m_3 = 0$: (a) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$; (b) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$; (c) $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\delta_{14} = \phi = \varphi = \delta = \sigma = 0$; (d) $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\delta_{14} = \phi = \varphi = \delta = \rho = 0$.

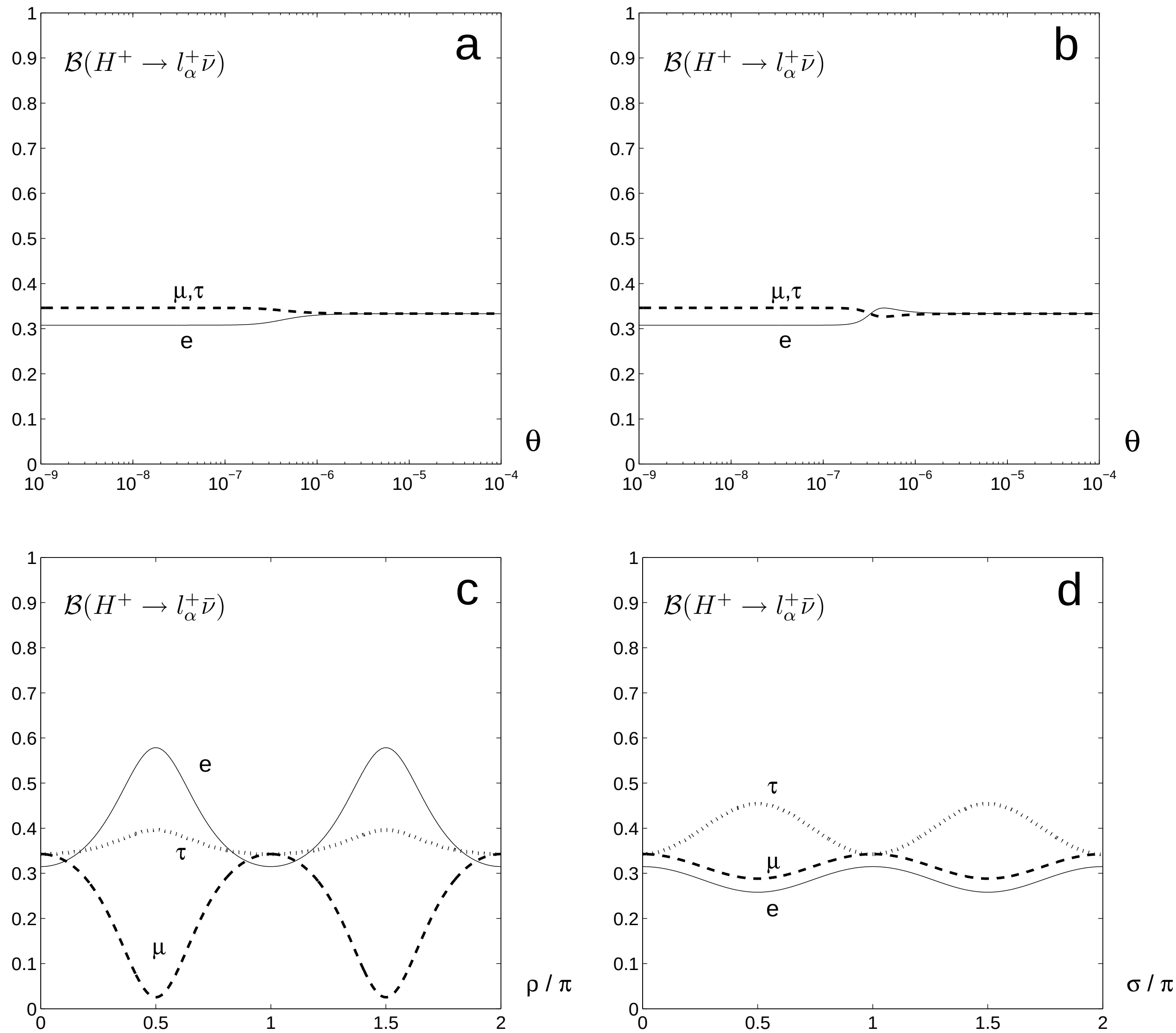

FIG. 3. Branching ratios of $H^+ \to l_\alpha^+ \bar{\nu}$ decays for the near degeneracy of m_i with $m_3 > m_2$: (a) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$; (b) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$; (c) $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\delta_{14} = \phi = \varphi = \delta = \sigma = 0$; (d) $\theta_{14} = \theta_{24} = \theta_{34} = 10^{-6.5}$ and $\delta_{14} = \phi = \varphi = \delta = \rho = 0$.